Differential Equations LaTeX Assignment

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Problem Statement

1. Find the general solution of the differential equation $y' - y\sin x = 0$.
2. Find the particular solution of the differential equation $y' - y\sin x = 0$ that satisfies the following initial condition:
   (a) $(0, -\frac{3}{e})$
   (b) $(0, \frac{1}{e})$
   (c) $(0, -\frac{1}{e})$
3. Sketch the slope field for $y' - y\sin x = 0$, along with the three solutions found in part (2).

Solution

1. The first step in solving the differential equation $y' - y\sin x = 0$ is to separate the variables.

$$\frac{dy}{dx} = y\sin x$$
$$\frac{1}{y}\,dy = \sin x\,dx$$

   Next, we integrate both sides of the equation with respect to the appropriate variable.

$$\int \frac{1}{y}\,dy = \int \sin x\,dx$$
$$\ln|y| = -\cos x + C_1$$

   Finally, we solve the equation for y in terms of x.

$$e^{\ln|y|} = e^{-\cos x + C_1}$$
$$|y| = e^{-\cos x} \cdot e^{C_1}$$
$$y = Ce^{-\cos x}$$

2. To find the particular solution, we substitute the initial condition into the general solution and solve for the constant C.

(a)

$$-\frac{3}{e} = Ce^{-\cos 0}$$
$$-\frac{3}{e} = Ce^{-1}$$
$$C = -3$$
$$\therefore y = -3e^{-\cos x}$$

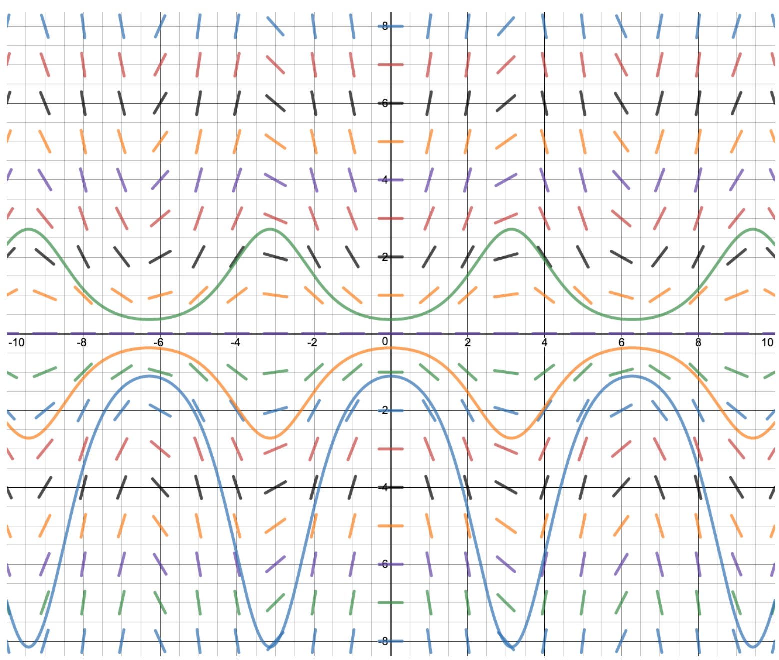
(b)

$$\frac{1}{e} = Ce^{-\cos 0}$$
$$\frac{1}{e} = Ce^{-1}$$
$$C = 1$$
$$\therefore y = e^{-\cos x}$$

(c)

$$-\frac{1}{e} = Ce^{-\cos 0}$$
$$-\frac{1}{e} = Ce^{-1}$$
$$C = -1$$
$$\therefore y = -e^{-\cos x}$$

3. The slope field for $y' - y \sin x = 0$ is shown below, along with the three solutions found in part (2).



```latex
\documentclass{article}

\usepackage[margin=1in]{geometry}
\usepackage{amsfonts, amsmath, amssymb}
\usepackage{graphicx}
\usepackage[none]{hyphenat}

\parindent 0ex %remove paragraph indent

\title{Differential Equations \LaTeX\ Assignment}
\author{Michelle Krummel}
\date{\today}

\setlength{\jot}{6pt}

\begin{document}
\maketitle

\section*{Problem Statement}
\begin{enumerate}
\item Find the general solution of the differential equation $y'-y\sin x=0$.
\item Find the particular solution of the differential equation $y'-y\sin x=0$
that satisfies the following initial condition:
\begin{enumerate}
\item $(0, -\frac{3}{e})$
\item $(0, \frac{1}{e})$
\item $(0,-\frac{1}{e})$
\end{enumerate}
\item Sketch the slope field for $y'-y\sin x=0$, along with the three solutions
found in part (2).
\end{enumerate}

\section*{Solution}
\begin{enumerate}
\item The first step in solving the differential equation $y'-y\sin x=0$ is to
separate the variables.
\begin{align*}
\frac{dy}{dx} &= y\sin x \\
\frac{1}{y}\,dy &= \sin x \,dx \\
\end{align*}

Next, we integrate both sides of the equation with respect to the appropriate
variable.
\begin{align*}
\int \frac{1}{y}\,dy &= \int \sin x \,dx \\
\ln|y| &= -\cos x+C_1 \\
\end{align*}

Finally, we solve the equation for $y$ in terms of $x$.
\begin{align*}
e^{\ln|y|} &= e^{-\cos x+C_1} \\
|y| &= e^{-\cos x}\cdot e^{C_1} \\
y &= Ce^{-\cos x} \\
\end{align*}
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\item To find the particular solution, we substitute the initial condition into the
general solution and solve for the constant $C$.
\begin{enumerate}
\item \begin{align*}
-\frac{3}{e} &=Ce^{-\cos 0} \\
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\end{align*}
\end{enumerate}

\item The slope field for $y'-y\sin x=0$ is shown below, along with the three
solutions found in part (2).\\[10pt]
\centering\includegraphics[width=4in]{sf1}

\end{enumerate}

\end{document}
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